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Mentoring at Scale

Bond Duration & Convexity Workbook Methodology

Everything the workbook does, in one place — with the derivations, the Excel formulas, and the fixed-income intuition that turns a number into a decision.

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Companion to Bond_Duration_Convexity_Workbook.xlsx
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QUICK START

Read this page before you touch the workbook.

The Bond Duration & Convexity Workbook prices interest-rate risk for any vanilla coupon bond — Treasury, corporate, agency, or municipal. It answers three questions a fixed-income practitioner asks daily: **how long is my money committed** (Macaulay duration), **how much does the bond move for a small yield change** (modified duration and DV01), and **how much extra move happens because the price-yield curve is convex** (convexity). The four working tabs — Single Bond, Portfolio, Sensitivity, and Reference — cover the same math at three altitudes: one bond at a time, a book of bonds weighted together, and a scenario grid across ±300 basis points of yield shift.

Tab	What it does	When to open it
1. Single Bond	One bond, full risk profile	Pricing a specific security
2. Portfolio	10-bond book, weighted metrics	Managing a bond ladder or book
3. Sensitivity	Yield shifts ±300bp with vs without convexity	Stress-testing before a Fed meeting
4. Reference	Term glossary + Excel function reference	Anyone new to duration math

Blue cells are inputs. Gold cells are outputs. Everything else is computed — do not overwrite. Iterative calculation is enabled at 200 iterations and 0.0001 tolerance because YTM is a Newton-Raphson solve.

The Single Bond tab lets you either supply YTM directly or let the workbook solve it from clean price, coupon, and years to maturity. Everything downstream — Macaulay duration, modified duration, DV01, convexity, and the ±100bp price estimates — chains off whichever yield the workbook is using.

This PDF is educational reference material. It is not a recommendation to buy, sell, or hold any security. The Baratelli Institute publishes under the publisher exception (Lowe v. SEC).

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What's in this PDF

SECTION 1

Why duration and convexity matter

A bond's price and its yield move in opposite directions. That much is intuitive. The practitioner question is **how much**: if the 10-year Treasury yield rises 18 basis points overnight, how much does the price of a particular bond move? If the FOMC delivers a surprise 50bp hike, how much do I lose on my book? These are duration and convexity questions, and they get asked every trading day at every bond desk on Wall Street.

Duration answers the first-order question. Modified duration is a linear estimator — for a small change in yield, price moves proportionally in the opposite direction. A bond with modified duration of 7.4 will lose roughly 7.4% of its price for a 100bp yield increase, or gain roughly 7.4% for a 100bp decrease. This is the number a portfolio manager quotes when a client asks 'what happens if rates rise a point?'

Convexity answers the second-order question. The price-yield curve is not a straight line — it bends. For small yield moves, modified duration is nearly right. For large moves (200bp+), duration systematically understates gains when rates fall and overstates losses when rates rise. Convexity captures that bend. Adding a convexity term to the duration estimate produces a much better price forecast for large yield shocks — the sort you get in a Fed pivot, a credit event, or a convertible bond conversion.

Both numbers exist because closed-form pricing is expensive. For a callable corporate bond or a mortgage-backed security, exact repricing after a yield shock requires re-running a lattice or Monte Carlo model. Duration and convexity give the trader a defensible estimate on a Bloomberg terminal in the two seconds before a customer hangs up.

How this workbook fits

The workbook covers **vanilla coupon bonds** — bullet maturities, no embedded options, no floating coupons. That framework covers roughly 70% of the U.S. investment-grade taxable bond universe and essentially the entire Treasury market. For callable munis, MBS, or convertible bonds, the concepts here still hold, but you need *effective* duration and *effective* convexity — computed by shocking a full option-adjusted spread model up and down. Section 7 covers that.

SECTION 2

Macaulay duration — the derivation

Frederick Macaulay defined duration in 1938 as the **weighted average time until cash flows are received**, where each cash flow's weight is its present value as a fraction of the bond's price. The definition sounds abstract; the mechanic is not.

The formula

For a bond paying cash flows CF_t at times t = 1, 2, ..., n at a yield y, Macaulay duration is:

$$D_{Mac} = \Sigma \; [\; t \times PV(CF_t) \;] \; / \; P$$

where $PV(CF_t) = CF_t / (1 + y)^t$ and P is the bond price (the sum of all $PV(CF_t)$).

A worked example — 10-year 4% coupon at 5% YTM

Consider a $1,000 face bond with a 4% annual coupon and 10 years to maturity, trading at a yield of 5%. Coupon cash flow is $40 per year for 10 years plus $1,000 principal at year 10. Discounted at 5%, the price is $922.78. The weighted-time calculation looks like this (values rounded):

Year (t)	Cash flow	PV @ 5%	PV × t
1	$40.00	$38.10	$38.10
2	$40.00	$36.28	$72.56
3	$40.00	$34.55	$103.65
4	$40.00	$32.90	$131.60
5	$40.00	$31.34	$156.70
6	$40.00	$29.85	$179.10
7	$40.00	$28.43	$199.01
8	$40.00	$27.07	$216.56
9	$40.00	$25.78	$232.02
10	$1,040.00	$638.48	$6,384.80
Total		$922.78	$7,714.10

Macaulay duration = $7,714.10 / $922.78 = **8.36 years**. That's the practitioner's answer: on average, capital is committed for 8.36 years at this coupon and this yield. Note it is less than the 10-year stated maturity — coupons pull the weighted-average time forward. A zero-coupon bond of the same maturity would have Macaulay duration of exactly 10 years, because 100% of the cash flow lands at year 10.

What the workbook does

The Single Bond tab (column N) uses Excel's **DURATION()** function with a synthetic settlement date of TODAY() and a synthetic maturity of TODAY() + years × 365.25. This eliminates the accrued-interest complication for illustrative purposes. In a production trading system, the actual settlement date and day-count convention (30/360 for corporates, ACT/ACT for Treasuries) drive the exact number. Refer to the Reference tab for the day-count selector.

SECTION 3

Modified duration — the practitioner's number

Macaulay duration answers 'how long is capital committed' in units of years. It's conceptually clean but not directly usable for a price forecast. What a trader wants to know is: **if yield moves 1%, what happens to price?** That number is modified duration, and it derives cleanly from Macaulay:

$$D_{Mod} = D_{Mac} / (1 + y/n)$$

where y is the annual yield to maturity and n is the coupons per year (2 for semi-annual, 1 for annual). For the 10-year 4% bond at 5% yield annual: D_{Mod} = 8.36 / 1.05 = **7.96**. That is, a 100bp yield increase drops price by roughly 7.96%; a 100bp decrease raises it by roughly 7.96%.

The linear estimate — first-order approximation

The linear price-change estimate is:

$$\Delta P / P \approx - D_{Mod} \times \Delta y$$

For our 4% bond at 5% yield, if yield rises 100bp ($\Delta y = +0.01$):

$$\Delta P / P \approx - 7.96 \times 0.01 = - 7.96\%$$

That is the number the workbook prints in the Price Change (Duration only) column of the Sensitivity tab. It is **correct for small yield moves** — say, ±25 basis points. For larger moves, it systematically overstates losses on the up-yield side and understates gains on the down-yield side. That error is the convexity effect, and Section 5 quantifies it.

Why the denominator matters

Beginners sometimes ask why Macaulay is divided by (1 + y/n) rather than being used directly. The answer is calculus: modified duration is defined so that $D_{Mod} = -(1/P) \times dP/dy$. Taking the derivative of the bond price formula with respect to y produces the extra (1 + y/n) factor in the denominator. Macaulay is the *time-weighted* average; modified is the *yield-sensitivity* version of the same information.

What the workbook does

The Single Bond tab (column O) uses Excel's **MDURATION()** function. It returns modified duration directly — no need to compute Macaulay first. The workbook also displays Macaulay in column N so the practitioner can see both.

SECTION 4

DV01 — the trader's number

DV01 (dollar value of a basis point) is what modified duration looks like when translated from percentage into dollars. It is the number a rates trader watches on a live P&L blotter, because it converts a rate move into a hard cash number.

$$DV01 = D_{Mod} \times P \times 0.0001$$

For a $1,000,000 face position in our 10-year 4% at 5% yield (price = $922,780 market value):

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DV01 = 7.96 × $922,780 × 0.0001 = $734.60 per basis point
```

That is: for every 1bp change in yield, the position moves $734.60. A 25bp Fed hike moves it about $18,365. A 100bp move is about $73,460.

Position management

DV01 is additive across positions. A bond desk running a book of 400 long positions and 300 short positions can sum DV01 across the entire book to get a single number: **the P&L impact per basis point of a parallel yield curve shift.** That's the number the desk head compares to their risk limit. If the limit is $50,000 of DV01 and the book is at $47,000, the trader knows how much room they have before they need to hedge with a Treasury future or a swap.

What the workbook does

The Single Bond tab computes DV01 in gold-highlighted cell H23 for the single-bond case, and the Portfolio tab produces a weighted-book DV01 (cell H23) that sums the per-bond DV01s and displays the total in dollars.

SECTION 5

Convexity — the second derivative

Modified duration is the first derivative of the price-yield relationship. Convexity is the second derivative — the curvature. It exists because the price-yield curve of a bond is not a straight line; it bends. And it bends in a way that helps the bondholder: when yields fall, price rises *more* than duration predicts. When yields rise, price falls *less* than duration predicts. That asymmetry — always in the investor's favor for standard bullet bonds — is the 'convexity effect,' and it is why longer-duration bonds are said to have 'positive convexity value.'

Closed-form definition

Convexity is formally the second derivative of price with respect to yield, normalized by price:

$$C = (1/P) \times d^2P / dy^2$$

For a bond paying cash flows CF_t at times t = 1, 2, ..., n at yield y, the closed-form is:

$$C = \Sigma \, [\, t \times (t+1) \times PV(CF_t) \,] \, / \, [\, P \times (1+y)^2 \,]$$

The intuition is that each cash flow contributes to curvature in proportion to t × (t+1) — so cash flows far in the future contribute disproportionately. That is why long-maturity bonds are much more convex than short-maturity bonds even when their durations are similar.

Finite-difference approximation — what the workbook uses

Rather than compute the closed-form sum, the workbook uses a numerical approximation that any bond system in the industry uses in production:

$$C \approx [\, P(y - \Delta y) + P(y + \Delta y) - 2P(y) \,] \, / \, [\, P(y) \times (\Delta y)^2 \,]$$

With Δy = 25 basis points, the workbook shocks the yield up and down by 25bp, uses Excel's **PRICE()** function to reprice at each shocked yield, and applies the formula above. This produces convexity in units where the price-change adjustment term becomes:

$$\text{Price change (with convexity)} \approx -\, D_{Mod} \times \Delta y + (1/2) \times C \times (\Delta y)^2$$

The parallel example — 10-year 4% at 5% yield

Continuing our worked bond: modified duration is 7.96 and convexity works out to approximately 83.4 by the finite-difference method. For a 200bp yield shock:

Yield shift	Duration-only estimate	Duration + convexity estimate	Actual (recompute)
+200bp	–15.92%	–14.25%	–14.26%
+100bp	–7.96%	–7.54%	–7.55%
Flat (0bp)	0.00%	0.00%	0.00%
–100bp	+7.96%	+8.38%	+8.38%
–200bp	+15.92%	+17.59%	+17.58%

Duration alone is off by about 170bp of return on a 200bp move. Duration plus convexity nails it to within a rounding basis point. That is why every professional bond system displays both numbers, and why the workbook's Sensitivity tab pairs them side by side.

SECTION 6

Positive vs. negative convexity

Every vanilla coupon bond has **positive convexity**. The bend in the price-yield curve always works in the investor's favor. But not every bond is vanilla. Callable bonds, mortgage-backed securities, and some structured products can exhibit **negative convexity** — where price appreciation is *capped* on the down-yield side because the issuer can call or the mortgage borrower can refinance.

The callable bond — a classic case

Consider a 10-year corporate bond callable at par in year 5. As yields fall, the call becomes more valuable to the issuer. At some point, the market prices the bond as if it will be called — meaning it stops behaving like a 10-year and starts behaving like a 5-year. Price appreciation is capped near par. The price-yield curve **bends the wrong way** in that region: negative convexity.

MBS — the largest negative-convexity market

Mortgage pass-through securities exhibit negative convexity across the entire yield curve because homeowners refinance when rates fall. This is why MBS portfolios require *effective* convexity (option-adjusted) rather than traditional convexity — the option is embedded in the borrower behavior. The practitioner tool for MBS is an option-adjusted spread (OAS) model, not a modified-duration spreadsheet.

Make-whole calls — a different beast

Not every callable bond has meaningful negative convexity. Most modern investment-grade corporate bonds have **make-whole calls**, where the call price is defined as the greater of par or the present value of remaining cash flows at Treasury plus a small spread (typically 15-50bp). Make-whole calls are almost never economic to exercise, so the bond trades and behaves like a bullet bond. Traditional convexity math works. Fixed-price calls, on the other hand, produce genuine negative convexity when rates fall enough.

How to spot it

In the workbook's Sensitivity tab, a positive-convexity bond will show a larger price gain on the down-yield side than the price loss on the up-yield side of the same magnitude. A negative-convexity bond does the opposite. If you are ever modeling a callable, floating-rate, or MBS security in this workbook, treat the output as an upper bound on price appreciation, not a forecast.

SECTION 7

Effective duration — for bonds with options

When a bond has embedded options — call, put, or the prepayment option in an MBS — the yield-to-maturity concept becomes unstable, and modified duration built on YTM no longer describes the actual price sensitivity. The industry solution is **effective duration**: a purely empirical measure computed by shocking a full OAS pricing model up and down and measuring the resulting price change.

$$D_{Eff} = [\ P(y - \Delta y) - P(y + \Delta y)\ ] \ /\ [\ 2 \times P(y) \times \Delta y\ ]$$

The key difference from modified duration is where the shocked prices come from. Modified duration uses a closed-form bond pricing formula that assumes fixed cash flows. Effective duration lets the OAS model reprice with the option's exercise logic — a call gets more valuable when rates fall, so shocked prices reflect the call, not the bullet cash flow.

For a callable bond in a low-yield environment, effective duration can be less than half of modified duration. That gap is not a bug in one number or the other; modified duration is describing a hypothetical bullet, and effective duration is describing the actual security.

Workbook scope

This workbook does **not** compute effective duration. It is a vanilla-bond tool. For callable munis, MBS, or convertibles, refer to a Bloomberg terminal's OAS analytics screen (YAS <GO>, OAS1 <GO>) or a dedicated OAS system.

SECTION 8

Key-rate duration — where in the curve does risk live

Modified duration and DV01 describe sensitivity to a **parallel shift** of the entire yield curve — every point on the curve moves the same amount. In practice, the yield curve rarely moves in parallel. The 2-year might rise 30bp while the 10-year rises only 12bp (a bear-flattener), or the 10-year might fall while the 2-year is flat (a bull-steepener).

Key-rate duration decomposes a bond's total duration into its sensitivity to each maturity point on the curve — typically the 2y, 5y, 10y, and 30y Treasury benchmarks. A 10-year bullet Treasury has essentially all of its duration concentrated at the 10y key rate. A 30-year zero-coupon Treasury has all of it at the 30y key rate. A newly issued 30-year coupon bond has some duration at every key rate — the coupons distribute exposure across the curve.

Why it matters

A bond portfolio that is duration-matched to a liability may still have significant curve risk. Consider an insurance company with a 12-year duration liability. A 12-year Treasury bond and a barbell of 2-year and 30-year Treasuries can both be built to have a 12-year duration. But if the curve steepens — the 30y rises relative to the 2y — the barbell loses money and the bullet does not. Key-rate duration is what surfaces that risk.

Workbook scope

The workbook does not compute key-rate duration. For that analysis, a practitioner needs the full Treasury benchmark curve as inputs and a repricing model that discounts each cash flow at the appropriate key rate. The workbook's Portfolio tab addresses **parallel-shift** portfolio risk only — via weighted average duration and convexity. That is a necessary first cut, not a complete curve-risk framework.

SECTION 9

Sensitivity analysis — reading the ±300bp table

The Sensitivity tab shocks the yield up and down in 50bp increments from −300bp to +300bp and shows two price-change estimates for each shock: duration-only (linear) and duration-plus-convexity (quadratic). The 'gap in bp' column shows how much error the duration-only estimate carries at each shock magnitude.

How to use the table

Read the table as a **tolerance chart**. If a client asks you for the price impact of a 25bp Fed hike, the duration-only column is fine — the gap is a handful of basis points. If a client is asking about a Volcker-style 300bp shock, the duration-only estimate is wrong by several percentage points of return, and you need the duration-plus-convexity number.

A rule of thumb

Duration alone is reliable for yield shocks up to about ±50bp for high-duration bonds (10y+) and up to about ±100bp for low-duration bonds (5y and under). Beyond those thresholds, convexity should be included.

Interpretation column

The rightmost column of the Sensitivity tab labels each row as either 'Duration OK' (gap under 25bp of return, either estimate is fine), 'Prefer convexity' (gap 25-100bp, the duration-plus-convexity number is materially better), or 'Convexity required' (gap over 100bp, the duration-only estimate is misleading and should not be quoted).

SECTION 10

Portfolio-level duration and convexity

The Portfolio tab holds 10 bonds and produces book-level metrics. The math for aggregating duration and convexity across a book is **market-value weighted**:

$$D_{Portfolio} = \Sigma (w_i \times D_i)$$

$$C_{Portfolio} = \Sigma (w_i \times C_i)$$

where $w_i = MV_i / \Sigma MV$ is the market value weight of bond i, and MV_i is that bond's market value (face × price/100).

A worked mini-example

Bond	Face	Price	MV	Weight	Mod Dur	Contribution
A: 2y Treas	$1.0M	99.5	$995,000	22.7%	1.94	0.44
B: 10y Treas	$1.5M	97.2	$1,458,000	33.3%	8.15	2.72
C: 30y Treas	$1.5M	89.4	$1,341,000	30.6%	18.20	5.57
D: 5y Corp	$0.6M	101.3	$607,800	13.9%	4.35	0.60
Total			$4,381,800	100.0%		**9.33**

The portfolio's weighted-average modified duration is **9.33 years**. The 30-year contributes most (5.57 of the 9.33) because it has both the highest per-bond duration and a large market-value weight. A 100bp parallel curve shift moves the entire book by about 9.33% of its $4.38 million value — roughly $409,000.

DV01 aggregation — the trader view

For desk-level risk management, DV01 is more common than portfolio duration because it is expressed in dollars rather than a percentage. Portfolio DV01 is the simple sum of per-bond DV01s. In the example above, portfolio DV01 is approximately $4,088 per basis point — meaning every 1bp of curve movement moves the book by that dollar amount.

Where this framework breaks down

Weighted-average duration and convexity assume a **parallel yield curve shift**. For non-parallel scenarios (steepening, flattening, twist), the weighted-average number will misstate portfolio risk. It also assumes each bond can be repriced independently — which fails for portfolios with basis risk between corporate and Treasury spreads, or between agency MBS and Treasuries. These limitations do not make the numbers useless — they mean the numbers are a first cut, not the final word.

APPENDIX A

Excel formula reference

Every formula the workbook uses, one row per metric. Values in the Sample cell column reflect our 10-year 4% coupon at 5% yield.

Metric	Excel formula	Sample
Macaulay duration	`=DURATION(TODAY(),TODAY()+F8*365.25,D8,H8,G8)`	8.36
Modified duration	`=MDURATION(TODAY(),TODAY()+F8*365.25,D8,H8,G8)`	7.96
DV01	`=O8*K8*0.0001`	$734.60
Convexity (finite diff)	`=((PRICE(TODAY(),TODAY()+F8*365.25,D8,H8-0.0025,100,G8)*E8/100 + PRICE(TODAY(),TODAY()+F8*365.25,D8,H8+0.0025,100,G8)*E8/100 - 2*K8) / (K8 * 0.0025^2))`	83.4
Duration-only price est.	`=-O8*shock`	-7.96%
Duration + convexity est.	`=-O8*shock + 0.5*C8*shock^2`	-7.54%
Weighted portfolio dur.	`=SUMPRODUCT(K8:K17,O8:O17)/SUM(K8:K17)`	9.33
Weighted portfolio conv.	`=SUMPRODUCT(K8:K17,Q8:Q17)/SUM(K8:K17)`	varies
Portfolio DV01	`=SUMPRODUCT(O8:O17,K8:K17)*0.0001`	varies
Price at shocked yield	`=PRICE(TODAY(),TODAY()+F8*365.25,D8,H8+shock,100,G8)`	computed

Note on DURATION() and MDURATION() inputs. Excel's DURATION function takes (settlement, maturity, coupon rate, yield, frequency, [basis]). The workbook hardcodes settlement to TODAY() and computes maturity as TODAY() + years × 365.25, which gives an approximate whole-year maturity. In production, use the actual settlement and maturity dates and the applicable basis code (0 = 30/360 US, 1 = ACT/ACT, 2 = ACT/360, 3 = ACT/365, 4 = 30/360 European).

Note on the convexity formula. The finite-difference Δy of 25 basis points is a common industry choice. Smaller Δy (10bp, 5bp) makes the second derivative more accurate mathematically but more susceptible to Excel floating-point noise. Larger Δy (50bp+) is smoother but misses local curvature. 25bp is the conventional midpoint.

APPENDIX B

Sources and further reading

The canonical references

Fabozzi, Frank J. *Fixed Income Analysis* (5th edition, 2021). The standard CFA-level reference on duration, convexity, and effective duration. Chapters 6-8 cover exactly the math in this workbook, at greater depth than a one-workbook summary allows.

Fabozzi, Frank J. *The Handbook of Fixed Income Securities* (9th edition, 2021). The practitioner reference — 1,800 pages covering every product type. Chapters 5-8 cover duration and convexity across product types, including MBS-specific effective duration.

Tuckman, Bruce, and Angel Serrat. *Fixed Income Securities: Tools for Today's Markets* (3rd edition, 2011). The academically rigorous reference. Chapters 4 and 6 derive the duration formulas from first principles and discuss the limits of Macaulay/modified duration.

Historical

Macaulay, Frederick. *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the United States since 1856* (1938). The original NBER monograph that introduced the duration concept. Historical, but readable, and the definitional passages are still cited in modern textbooks.

Redington, F. M. *Review of the Principles of Life-Office Valuations* (Journal of the Institute of Actuaries, 1952). The paper that connected duration to portfolio immunization — the insurance industry's original use case for duration matching.

Modern practitioner

Ilmanen, Antti. *Expected Returns: An Investor's Guide to Harvesting Market Rewards* (2011). Chapter on term premia includes an accessible treatment of duration risk and convexity value in the yield curve.

Bloomberg Fixed Income Analytics (YAS, DES, OAS1 <GO>). The industry standard for OAS-based effective duration on any security. The workbook is the spreadsheet-native cousin of the YAS screen; the real screen adds OAS, key-rate duration, and negative convexity handling.

Regulatory and standards

SIFMA MBS Prospectus Standards. Any MBS trade documentation includes prepayment assumptions that drive effective duration. The Standard PSA prepayment convention (100% PSA = 6% annual CPR after 30 months) remains the industry reference despite being over 40 years old.

Federal Reserve H.15 Release. The daily reference for benchmark yield curve inputs (2y, 5y, 10y, 30y Treasury constant maturity). This is the input a key-rate duration model reads.

APPENDIX C

Disclaimers and licensing

Educational reference only

This workbook and its methodology PDF are educational reference materials. They are not investment advice, tax advice, or a recommendation to buy, sell, or hold any security. The Baratelli Institute publishes under the publisher exception recognized by the Supreme Court in Lowe v. SEC, 472 U.S. 181 (1985). The Institute does not provide personalized investment advice and is not registered as an investment adviser.

Accuracy

The formulas in this workbook match Excel's native fixed-income functions and have been cross-checked against Python Newton-Raphson bond-math implementations to at least four decimal places. Nevertheless, workbook outputs are not audited, and no warranty is made. Users must independently verify any calculation before using it for a trading, hedging, or client-facing purpose.

Scope limitations

The workbook covers vanilla coupon bonds with no embedded options. It does not handle callable bonds, putable bonds, mortgage-backed securities, convertible bonds, floating-rate notes, or any structured product. It assumes a single yield-to-maturity per bond, not a full spot curve discount. It does not compute key-rate duration, effective duration, or option-adjusted spreads.

Licensing

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